## Exercise 58

The function $f(x) = \sin(x + \sin 2x)$, $0 \leq x \leq \pi$, arises in applications to frequency modulation (FM) synthesis.

(a) Use a graph of f produced by a calculator to make a rough sketch of the graph of f'.

(b) Calculate $f'(x)$ and use this expression, with a calculator, to graph f'. Compare with your sketch in part (a).

Solution

Take a derivative of the given function.

$$f'(x) = \frac{df}{dx} = \frac{d}{dx}\left[\sin(x + \sin 2x)\right]$$

$$= \cos(x + \sin 2x) \cdot \frac{d}{dx}(x + \sin 2x)$$

$$= \cos(x + \sin 2x) \cdot \left[1 + (\cos 2x) \cdot \frac{d}{dx}(2x)\right]$$

$$= \cos(x + \sin 2x) \cdot \left[1 + (\cos 2x) \cdot (2)\right]$$

$$= \cos(x + \sin 2x)(1 + 2\cos 2x)$$
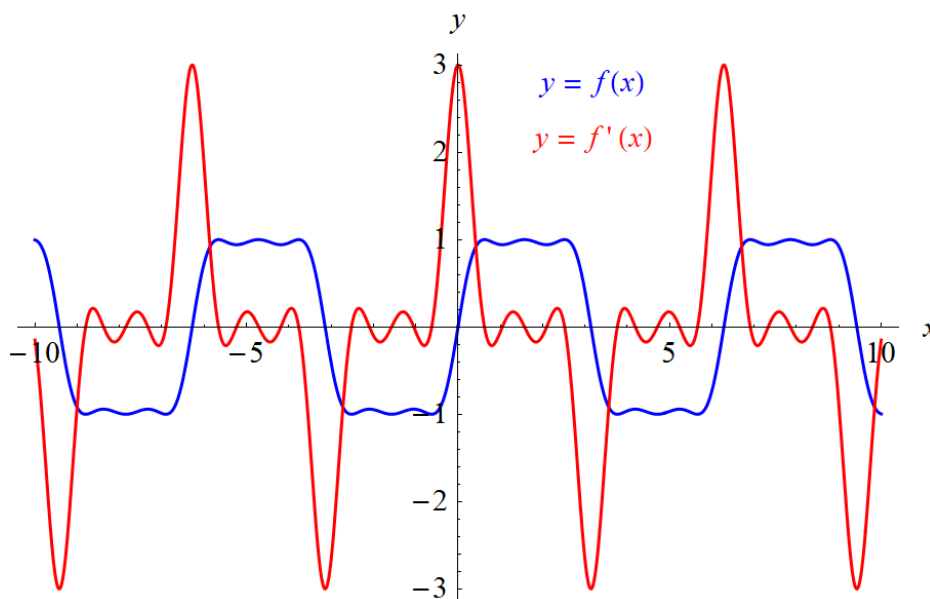
The graph of $f'(x)$ is negative wherever $f(x)$ is decreasing, and the graph of $f'(x)$ is positive wherever $f(x)$ is increasing.